Mathematics 434 Spring 2011 Problem Sheet 8

Due March 28

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

"*The shortest path between two truths in the real domain passes through the complex domain.*" – Jacques Hadamard

"*Such is the advantage of a well-constructed language that its simplified notation often becomes the source of profound theories.*" – P. S. Laplace

Problems

1. Do **two** of the following.
   (a) Let R be an integral domain. Prove the polynomial ring $R[x]$ is also an integral domain.
   (b) Let R be an integral domain. Prove the invertible elements of the polynomial ring $R[x]$ are the units of R.
   (c) An integral domain with finitely many elements is a field.
2. Prove the maximal ideals in the ring of integers are the principal ideals generated by prime integers.
3. Determine the maximal ideals of $\mathbf{R}[x] / (x^2 - 3x + 2)$ where $\mathbf{R}$ denotes the real numbers.
4. Let R be a ring and let I be an ideal of R. Let M be an ideal of R containing I and let $\overline{M} = M/I$ be the corresponding ideal of $\overline{R}$. Prove M is maximal if and only if $\overline{M}$ is maximal.
5. Prove either of the following:
   (a) $\mathbf{Z}_2[x] / (x^3 + x + 1)$ is a field.
   (b) $\mathbf{Z}_3[x] / (x^3 + x + 1)$ is not a field.